PHY 481/581 - Homework Set 3

Northern Arizona University Due: 10/15/2018

1 1D Monatomic Solid

Problem 1. Derive the dispersion relation for the monatomic 1D solid. Utilize the equation of motion for the atom located at equilibrium position $x_n = na$, where a is the spacing between atoms at equilibrium (unit cell), given by

$$M\ddot{u}_n = -\alpha\big[2u_n - u_{n+1} - u_{n-1}\big] \tag{1}$$

where u_n is the displacement from equilibrium for atom n and α is the "spring constant." Hint: start with the solution $u_n = Ae^{i(kx_n - \omega t)}$ and note that $x_{n+1} = a(n+1)$, *etc*.

2 1D Diatomic Solid

Problem 2. Derive the dispersion relation for the longitudinal oscillations of a one-dimensional diatomic mass-and-spring crystal where the unit cell is of length a and each unit cell contains one atom of mass m_1 and one atom of mass m_2 connected together by springs with spring constant α.

Problem 3. Determine the frequencies of the acoustic and optical modes at $k = 0$ as well as at the Brillouin zone boundary.

Problem 4. Sketch the dispersion in both reduced and extended zone scheme.